Econ 802

First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Define the production possibilities set Y so that $y_1 \leq 0$ and $y_2 \leq 0$ are inputs and $y_3 \geq 0$ is an output. Let $y = (y_1, y_2, y_3)$ be feasible if and only if $0 \leq y_3 \leq (y_1 + y_2)^2$.

(a) Construct a graph of a typical input requirement set $V(y_3)$. Is this set non-empty? monotonic? closed? bounded? convex? strictly convex? Explain carefully.

(b) Given this technology, would you expect the firm's cost minimization problem to have a solution? If so, describe the solution. If not, explain why not.

(c) Given this technology, would you expect the firm's profit maximization problem to have a solution? If so, describe the solution. If not, explain why not.

2. For the following production functions, let $(x_1, x_2) \geq 0$ with $a > 0$ and $b > 0$.
   (i) $y = ax_1 + bx_2$
   (ii) $y = x_1^a x_2^b$
   (iii) $y = \min \{ax_1; bx_2\}$

(a) Using whatever methods seem most appropriate, solve for the conditional input demands $x_i(w,y)$ for i = 1, 2 in each case and carefully justify your answers.

(b) Suppose an undergraduate student knows about partial derivatives and Lagrange multipliers, but has not taken Econ 802. For which of the cases would the student be likely to get the <u>right</u> answers in (a)? For which of the cases would the student be likely to get the <u>wrong</u> answers in (a)? Explain.

(c) "Everybody knows that if the wage rate goes up, firms will spend a larger share of total cost on labor." For which of the cases (i), (ii) and (iii) is this true? Use math and also give some verbal intuition for your results.

3. Write a typical production plan as $y = (y_1 \,.\,.\, y_n)$, where positive entries are outputs and negative entries are inputs. Write the price vector as $p = (p_1 \,.\,.\, p_n)$, where all prices are strictly positive. Let Y be the production possibilities set. Define the profit function to be $\pi(p) =$ max py subject to $y \in Y$. Assume this function is well-defined for all relevant price vectors.

(a) Prove that $\pi(p)$ is non-decreasing in p. Do not use calculus. Then give a short intuitive explanation for this result.

(b) Prove that $\pi(p)$ is linearly homogeneous in p. Do not use calculus. Then give a short intuitive explanation for this result.

(c) Prove that $\pi(p)$ is convex in p. Do not use calculus. Then give a short intuitive explanation for this result.

4. Throughout this question, assume the production function has constant returns to scale and is strictly quasi-concave. Also assume all functions are differentiable.

(a) Prove that the conditional input demand functions can be written in the form $x_i(w, y) = yz_i(w)$ for all $i = 1 \ldots n$, where $z_i(w)$ is a function that depends on the price vector $w = (w_1 \ldots w_n)$ but <u>not</u> the output level y.

(b) For the two-input case, fix the price vector $w = (w_1, w_2)$. Define the expansion path to be the set $\{(x_1, x_2) \geq 0$ such that (x_1, x_2) minimizes $w_1x_1 + w_2x_2$ subject to $f(x_1, x_2) = y$ for some $y \geq 0\}$. Thus, with input prices held constant, a point (x_1, x_2) is on the expansion path if there is some output level at which that point solves the cost minimization problem. Using the result from part (a), draw a graph of the expansion path for a fixed price vector, and explain what the result in (a) implies about the behavior of the technical rate of substitution as y increases.

(c) For the two-input case, fix the price vector (w_1, w_2). Define the long run cost function c(y) in the usual way, and let the short run cost function be $c(y, x_2)$ where the level of x_2 is fixed in the short run but x_1 is variable. Draw a graph showing the long run average cost curve, as well as the short run average cost curve for some fixed $x_2 > 0$. Prove that there is one and only one level of output at which these curves touch. If possible, relate your proof to your analysis in part (b).

5. Here are some miscellaneous questions.

(a) "If the production possibilities set Y is convex but not strictly convex, then the firm's profit maximization problem has a solution but the solution is not unique". Discuss this statement and use a graph to explain your reasoning.

(b) State the weak axiom of profit maximization (WAPM) and show how it can be used to obtain predictions about firm behavior.

(c) Assume the Hessian matrix of the production function is negative definite at all points. Use the first order conditions for profit maximization to solve for the substitution matrix $\partial x/\partial w$ (note that the question is asking about unconditional factor demands). What important properties does $\partial x/\partial w$ have? Why?